

Goal Assignment using Distance Cost in Multi-Robot Exploration

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Abstract—In this paper, we discuss the problem of goal assignment in the multi-robot exploration task. The presented work is focused on the underlying optimal assignment problem of the multi-robot task allocation that is addressed by three state-of-the-art approaches. In addition, we propose a novel exploration strategy considering allocation of all current goals (not only immediate goal) for each robot, which leads to the multiple traveling salesman problem formulation. Although the problem is strongly NP-hard, we show its approximate solution is computationally feasible and its overall requirements are competitive to the previous approaches. The proposed approach and three well-known approaches are compared in series of problems considering various numbers of robots and sensor ranges. Based on the evaluation of the results the proposed exploration strategy provides shorter exploration times than the former approaches.

I. INTRODUCTION

The mobile robot exploration is a complex task in which a mobile robot is autonomously navigated in an unknown environment in order to create a map of the environment. The exploration can be defined as an iterative process determining a new goal for the robot and its navigation towards the goal. The process is terminated whenever a complete map of the environment is created. Having a team of robots, an efficient allocation of exploration targets among the team is a natural way how to reduce the required time to collect information about an unknown environment.

The problem of Multi-Robot Exploration (MRE) is a kind of the Multi-Robot Task Allocation (MRTA) [1] in which tasks are new goal locations towards which robots are navigated. The fundamental way how to determine candidates for goal locations is the frontier based approach proposed by Yamauchi in 1998 [2] and further extended by many researchers later, e.g., see one of the recent work [3].

Having a set of candidate positions the robot's next goal can be determined regarding a selected criteria. A unifying concept of how to evaluate candidate positions is based on the goal utility. Although various utility functions have been proposed, all of them basically combine information gain (or expected benefit [4]) together with the required travelling distance to the goal [5]. Then, the robot's next goal is repeatably selected from goal candidates. Such an assignment of the next robot goal is called a next-best-view approach and it represents the fundamental stream in exploration [6].

The next-best-view approach can be formulated as the optimal assignment problem studied in operational research [1].

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The problem is to find the best assignment of n goals to m robots maximizing the overall utility, i.e., to find one goal for each robot. The problem can be solved in polynomial time using the Hungarian algorithm. The algorithm has been applied to MRE in [7], where authors use Voronoi Graphs of the current known environment to explore a single room by one robot.

A distributed assignment algorithm called *Broadcast of Local Eligibility* (BLE) has been proposed in [8]. A pair $\langle robot, task \rangle$ with the highest utility is considered to assign the task to the robot without tasks. The BLE algorithm works iteratively until each robot has assigned a task; thus, the algorithm is also called iterative assignment.

Another stream of distributed MRE solutions is based on market (or auction) based approaches in which a robot (auctioneer) offers a task and other robots bid. If any robot bids with a higher price than the auctioneer's offer, the task is exchanged. This approach is used in [9], where a robot considers its goals in a tour and new (exchanged) goal is inserted into the tour regarding minimization of the tour's cost, i.e., the problem is a variant of the traveling salesman problem (TSP).

A selection of the next navigational goal considering the TSP distance cost has been studied in our previous work [10]. The cost is computed as the length of the shortest path connecting the robot with the candidate goal and all remaining goals. The Chained Lin-Kernighan heuristic [11] is considered to find a solution of the TSP, which provides sufficiently good solution without expensive computational requirements. Considering visitation of all current goals leads to about 30 percentage points shorter exploration path than using the standard greedy approach.

In this paper, we examine the TSP approach in MRE as the multiple traveling salesman problem (MTSP). The encouraging results presented in [10] motivate us to consider similar approach also in MRE; however, here, the key issue is how to determine and assign particular set of all goals to each robot in order to compute the distance cost as the length of the tour visiting all goals in the set using a solution of the related TSP. We propose to cluster the goals into m clusters first, where m is the number of robots. After that, a $\langle cluster, robot \rangle$ pair is evaluated using the TSP distance cost [10] for determining the next robot's goal.

The proposed approach is similar in the TSP aspect with the approach [9]. The main difference is that our approach is focused on the explicit MTSP formulation and the proposed solution is compared with the greedy approach [12], iterative assignment [8], and the Hungarian algorithm [7], which (to the best of our knowledge) has not been published yet.

Moreover, during an experimental verification of the tested approaches we have found out that the studied performance metric significantly depends on particular components of the whole navigational system, especially on a local path planner. Therefore, inspired by the methodology described in [13] we designed a multi-robot exploration framework in which we can isolate the assignment problem and fix the navigation issues. In consequence, the framework provides same conditions for all evaluated methods during the whole exploration process. However, it is clear that real benefits of the exploration strategy should be verified in real experiments. Therefore, the methods have been also evaluated in selected problems using the Player/Stage framework [14], like in the aforementioned approaches.

The remainder of the paper is organized as follows. The problem statement is presented in Section II and a brief description of the examined methods in Section III. In Section IV, the MRE framework used in the evaluation is described. The proposed solution of the assignment problem based on the MTSP formulation is presented in Section V. Evaluation of the results and discussions of the MRE issues are presented in Section VI. Finally, Section VII is dedicated to concluding remarks.

II. PROBLEM STATEMENT

Although the evaluated approaches are general and not necessarily restricted to the particular sensors or map building techniques, we consider laser range finder sensor and occupancy grid approach for building a map of the unknown environment. The addressed problem of the multi-robot exploration (MRE) stands for building a map of the unknown environment using a team of m identical robots equipped with a laser range finder. The map \mathcal{M} is formed from the occupancy grid using threshold values for probability that the grid's cell is occupied or free [2]. Thus, a cell in the navigational grid represents freespace, obstacle, or unknown part of the environment.

The exploration algorithm is an iterative procedure that is terminated once the navigational grid does not contain a reachable cell with an unknown value. At each exploration step, the robots' goals are determined from a set of candidate positions that are found as representatives of frontier cells.

The goals are assigned to robots using the exploration strategy that can be formalized as follows. *Let the current n goals be located at positions $\mathbf{G} = \{g_1, \dots, g_n\}$ and the current robot poses be $\mathbf{R} = \{r_1, \dots, r_m\}$. The problem is to determine a goal $g \in \mathbf{G}$ for each robot $r \in \mathbf{R}$ that will minimize the total required time, which can be approximated by the maximal travelled distance by an individual robot, to explore the whole environment.* The assignment is performed according to the particular strategy using defined utility and cost functions. In this paper, we consider only a distance cost \mathcal{L} for evaluating the goal assignment; however, the examined assignment strategies are general and can also be used with a combined value of distance and utility costs.

For the standard strategies (described bellow) the distance cost $\mathcal{L}(g_i, r_j)$ (where $g_i \in \mathbf{G}$ and $r_j \in \mathbf{R}$) is the length of

the shortest collision free path from the robot r_j to the goal g_i , e.g., found by the Distance Transform algorithm [15]. The proposed MTSP based assignment strategy utilizes the TSP distance cost [10].

In this paper, we consider the total distance travelled by a robot as the performance metric. The main motivation for utilizing several robots is expected reduction of the total required time to explore the whole environment, therefore we are looking for the maximal distance travelled as short as possible. Thus, having m robots with the distances travelled l_1, l_2, \dots, l_m the distance metric is $L = \max\{l_1, l_2, \dots, l_m\}$.

III. STANDARD GOAL ASSIGNMENT STRATEGIES

Greedy Assignment – The greedy assignment is based on the approach proposed by Yamauchi in [12]; however, it is modified to avoid assignment of the same goal to two robots because a centralized approach is considered here. The modification is that a random permutation of the robots $\Pi(\mathbf{R})$ is created first. Then, for each robot from $r \in \Pi(\mathbf{R})$ the best not assigned goal from \mathbf{G} is found. The complexity of this assignment algorithm can be bounded by $O(nm)$.

Iterative Assignment – The iterative assignment follows the BLE algorithm [8], but for simplicity it is also implemented in a centralized manner. First, all robot-goal pairs $p = \langle r, g \rangle$ are created and ordered using the distance cost \mathcal{L} , i.e., $\mathcal{L}(p_1) \leq \mathcal{L}(p_2), \dots \leq \mathcal{L}(p_l)$. After that, the ordered sequence is traversed starting from its first element, and the first not already used goal is iteratively assigned to a robot without the goal. The complexity of the iterative assignment can be bounded by $O((nm) \log(nm))$.

Hungarian Method – The Hungarian algorithm provides the optimal assignment of the n goals to m robots with the time complexity $O(n^3)$ for $n \geq m$. Similarly to the iterative assignment the cost matrix is determined using the distance cost \mathcal{L} , where rows stand for robots and columns for goals. In particular we consider the C implementation of the algorithm developed by Cyrill Stachniss [16].

IV. MULTI-ROBOT EXPLORATION FRAMEWORK

Similarly to the approach [13] we consider a simulator for a focused investigation of exploration strategies. The simulator is based on a grid map that provides discrete timing of navigation and sensing operations. In particular, the motion consists of independent turning and moving steps using the grid cells (e.g., a robot visits all grid cells during its motion along a straight line segment) while the sensing is performed at each such a motion step. The framework also allows to easily switch the simulator with some robot control framework like Player/Stage or ROS; thus, MRE strategies developed can be easily deployed to control real robots.

A schema of the exploration loop is depicted in Algorithm 1. First, the initial robots surroundings are sensed and the occupancy grid is updated accordingly (Line 3). Then, the navigation grid is created from the occupancy grid and all frontiers are detected. The frontiers are particular grid cells (a set of freespace cells that are incident with cells with the unknown value using 8-neighbourhood) that form a

Algorithm 1: MRE Framework

Input: \mathbf{R} - a set of m robots**Input:** s_{max} - the maximal number of the performed navigation steps before new assignment**Output:** \mathcal{M} - a map of the explored environment

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1 Initialization of the occupancy grid  $Occ$ 
2  $\mathcal{M} = \emptyset$  // the environment is unknown
3 foreach  $r_j \in \mathbf{R}$  do update( $Occ$ , robot_sense( $r_j$ ))
4 repeat
5    $\mathcal{M} = \text{create\_navigation\_grid}(Occ)$ 
6    $\mathbf{F} = \text{detect\_all\_frontiers}(\mathcal{M})$ 
7    $\mathbf{G} = \text{determine\_representatives}(\mathbf{F})$ 
8    $(\langle r_1, g_{r_1} \rangle, \dots, \langle r_m, g_{r_m} \rangle) = \text{assign\_goal}(\mathbf{R}, \mathbf{G}, \mathcal{M})$ 
9   fix_assignment( $\mathbf{G}$ ,  $\langle r_1, g_{r_1} \rangle, \dots, \langle r_m, g_{r_m} \rangle$ )
10  Create navigation plan  $\mathbf{P}_i$  for each pair  $\langle r_i, g_{r_i} \rangle$ .
11   $l = \min\{|\mathbf{P}_1|, \dots, |\mathbf{P}_m|\}$  // the plan length
12   $k = \min\{l, s_{max}\}$ 
13  for  $i = 1..k$  do
14    foreach  $r_j \in \mathbf{R}$  do
15      move( $r_j$ ,  $\mathbf{P}_j(i)$ )
16      update( $Occ$ , robot_sense( $r_j$ ))
17 until  $|\mathbf{G}| == 0$ 
```

set of connected components. The goals for the assignment are found as representatives of the connected components using the K-means algorithm. The number of representatives n_r of a single component F with $f = |F|$ frontier cells is determined as

$$n_r = 1 + \left\lfloor \frac{f}{1.8D} + 0.5 \right\rfloor, \quad (1)$$

where D is the sensor range (in grid cells). A detailed description of the selection procedure can be found in [10].

Once the goals are determined the selected exploration strategy (Line 8) is used to assign a goal to each robot. It may happen that in the resulting assignment a robot can be without the associated goal, e.g., for $|\mathbf{G}| < m$. In such a case, the assignment is fixed (Line 9) using the closest goal to the robot, because it is desirable to utilize all the robots for the whole exploration period in order to minimize the total required time of the exploration (here, we assume the explored environment is a single connected component).

The assigned goals are used to determine the execution plan consisting of simple operations. The plan is then executed up to s_{max} steps (Lines 13–16). This part of the loop is replaced by adding goals to a local path planner if the algorithm is used with real navigation system, e.g., using the Player framework. Finally, the exploration loop is terminated if all reachable parts of the environment are explored.

It should be noted that the set \mathbf{G} contains only representatives that are reachable by at least one robot, i.e., a collision free path exists in \mathcal{M} . The paths are found using the Distance Transform algorithm [15] that are then simplified by a greedy ray-shooting method using Bresenham's algorithm. The simplification does not affect the length of the path (on a grid) but the path is smoother.

V. PROPOSED MTSP BASED ASSIGNMENT

The proposed exploration strategy is based on formulation of the goals' assignment problem as the MTSP. Having the set of goals \mathbf{G} and m robots at positions $r_i \in \mathbf{R}$ for $i = 1..m$, the problem is to find m tours starting at the robots positions r_i such that each goal $g \in \mathbf{G}$ is contained in at least one tour and the length of the longest tour is minimal. It is known the MTSP problem is NP-hard, and therefore, we consider approximate solution of the problem based on an assignment of m clusters of the goals to robots, i.e., a kind of cluster-first, route-second heuristic approach. The proposed MTSP based assignment can be summarized in the following steps.

- 1) Find m clusters $\mathbf{C} = \{C_1, \dots, C_m\}$, where $C_i \subseteq \mathbf{G}$.
- 2) Determine the TSP distance cost for each pair $\langle C_i, r_i \rangle$, where $C_i \in \mathbf{C}$ and $r_i \in \mathbf{R}$.
- 3) Extract the first goal $g \in \mathbf{G}$ of the TSP tour from each non-empty cluster C_i assigned to the robot r_i .
- 4) Fix goals' assignment if there is an empty set C_i .

Although the proposed clustering based solution of the MTSP is fairly common, we suggest (regarding the context of MRE) the following particular solutions of the clustering and a direct assignment of the clusters to the robots.

A. Goals Clustering

Various methods of clustering can be used. One of the popular algorithms is K-means; however, a regular variant of this algorithm is based on the Euclidean distance between samples. The distances between goals on frontiers in the map of the environment being explored are rather geodesic due to presence of obstacles (or missing information). Therefore, using the Euclidean distance provides clusters for which real paths to the goals are significantly longer than the expected.

Regarding this fact a more general variant of the K-means algorithm can be used, e.g., [17]. Alternatively, goals can be transformed to the Euclidean space where their mutual distances are preserved using SAMCOF (Scaling by Maximizing a Convex Function) [18]. Then, a regular K-means algorithm can be used. In this paper, we consider

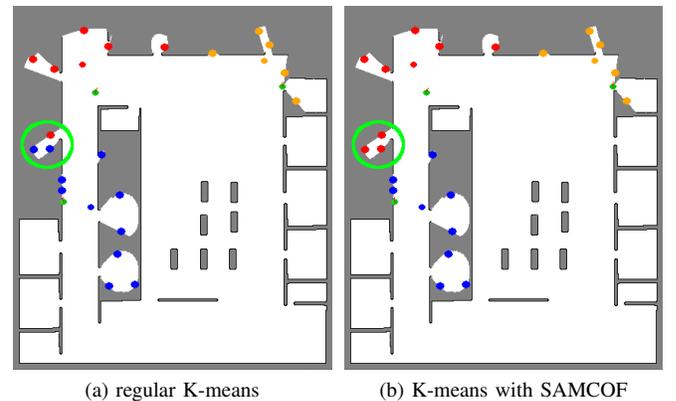


Fig. 1. An example of the found clusters within the jh environment using a regular K-means algorithm on goals and transformed goals. The goals in the clusters are shown as red, blue, and orange disks. Unknown parts of the environment are in gray. The current robot positions are shown as green disks and the green circle highlights the effect of SAMCOF utilization.

the SAMCOF transformation and an example of the found clusters can be seen in Fig. 1. The K-means algorithm is initialized using position of the robots, and therefore, also the robots' positions are transformed using SAMCOF as well.

B. Fixing Goals' Assignment

The utilized initialization of the K-means algorithm results to the clustering where a cluster C_i is formed in the vicinity of r_i . Thus, the clustering prefers assignment of goals that are close to the robot, which follows the greedy strategy that is advantageous in the case of separable clusters and robots faraway each other. On the other hand, it may happen that two robots are close, and therefore, one cluster dominates over the other, which results in a situation when all the goals are in the first cluster and the second cluster is empty. In such a situation, it is important that the robot with an empty cluster is moved towards unexplored part of the environment. Otherwise (e.g., when the robots stops its motion), it will be more and more far from new goals, which may result the robot will not actively participate on the exploration.

Various strategies how to determine a goal for a robot with an empty cluster can be proposed. Regarding the considered TSP distance cost, we proposed to assign a goal according to the expected time when the goal will be visited. Therefore, once a tour visiting all goals in the cluster is determined for each robot r_i with $C_i > 0$, a length of the path from the particular robot to the goal (along the tour) denotes the expected time of visit. Then, the goals are ordered using the time and goals with higher times are sequentially assigned to the robots without already assigned goal.

This procedure assigns a goal to each robot, and therefore, it replaces the `fix_assignment` in Algorithm 1 (Line 9).

VI. RESULTS

Three standard approaches and the proposed MTSP based approach have been evaluated in the developed MRE framework first. The framework allows focused study of exploration strategies that can be fully controlled by the trial setup. Herein presented experimental evaluation has been performed using a map of the environment (called *jh*) representing a real administrative building with dimensions 21 m×24 m. The environment is large enough to exhibit performance of the MRE using several robots while it also contains cycles and long corridors with several rooms. Thus, it provides representative office-like environment for verifying feasibility of the proposed MTSP strategy.

We followed recommendations of benchmarking the exploration strategies presented in [13] and considered small perturbations in the initial positions of the robots forming 20 variants of each problem defined by the sensor range ρ , the number of robots m , and the maximal planning period given by s_{max} . The iterative assignment and Hungarian exploration strategies are completely deterministic, while the Greedy and the proposed MTSP methods are stochastic. Therefore for each problem variant a single trial is considered for the deterministic ones and 20 trials are performed for the stochastic methods. The studied performance metric is then

computed over all perturbations (problem variants) and trials as average values (denoted as L) and standard deviations (s_L). All presented distance values are in meters.

The used sensor is a laser range finder HOKUYO with 270° field of view. The occupancy and navigational grids (map) have identical dimensions with the cell size 0.05 m×0.05 m.

A. Comparison of the Assignment Strategies

The exploration strategies are compared using $s_{max}=7$ that provides a good trade-off between the quality of solution and computational requirements, see Fig. 3. The considered numbers of robots are $m \in \{3, 5, 7, 10\}$ and the sensor range is selected from the set $\rho \in \{3, 4, 5\}$ meters, which results in 10 080 trials in total for this evaluation.

TABLE I
MAXIMAL TRAVELLED DISTANCE, $s_{max}=7$

ρ	m	Greedy		Iterative		Hungarian		MTSP	
		L	s_L	L	s_L	L	s_L	L	s_L
3.0	3	94.8	18.9	81.9	8.1	81.4	6.6	69.6	4.3
3.0	5	68.0	9.4	55.6	5.3	56.1	4.5	47.7	3.3
3.0	7	58.2	6.4	50.4	3.0	49.7	3.0	44.5	3.0
3.0	10	68.8	12.4	40.7	1.8	37.9	1.4	42.5	2.4
4.0	3	90.4	15.4	74.9	6.6	77.0	4.7	58.9	3.5
4.0	5	85.2	52.3	53.6	8.0	48.5	4.0	46.2	3.3
4.0	7	77.2	42.0	49.3	5.7	47.5	3.9	44.1	3.4
4.0	10	68.0	13.7	40.0	1.6	37.1	1.6	40.6	2.8
5.0	3	72.2	8.8	66.9	6.3	65.1	2.5	54.6	1.4
5.0	5	70.7	6.8	58.6	3.1	56.8	3.2	45.4	2.1
5.0	7	68.6	8.7	51.7	3.0	49.6	2.8	43.2	3.1
5.0	10	64.2	13.4	39.7	1.6	37.1	1.2	40.9	2.5

The results are shown in Fig. 2 and detailed results in Table I. The MTSP provides shorter exploration paths than other strategies; however, with increasing m the benefits of the MTSP strategy is not evident from the average values. Therefore we performed statistical evaluation using a null hypothesis that the algorithms provide statistically identical results. We consider the Wilcoxon test for the evaluation, because we assume the distributions are not Gaussian (based on the Shapiro-Wilk test).

The strategies are considered different if the P-values obtained by the Wilcoxon test are less than 0.001, which indicates the difference between L is statistically significant and a strategy providing lower L is considered as providing better results. Results of the statistical comparison are shown in Table II. All the p-values are very small, therefore characters '-', '+', and '=' are used to denote that the particular strategy provides longer, shorter or statistically identical L .

Regarding the results the considered range does not significantly affect L because of relatively small open parts while the rooms must be explicitly visited. Notice the standard deviation s_L for the greedy strategy. It indicates the performance is varying and sometimes the solution can be very close to the solution found by the MTSP. However, in average, it is worse than all other strategies. Although, the Greedy strategy is stochastic, we found that the performance

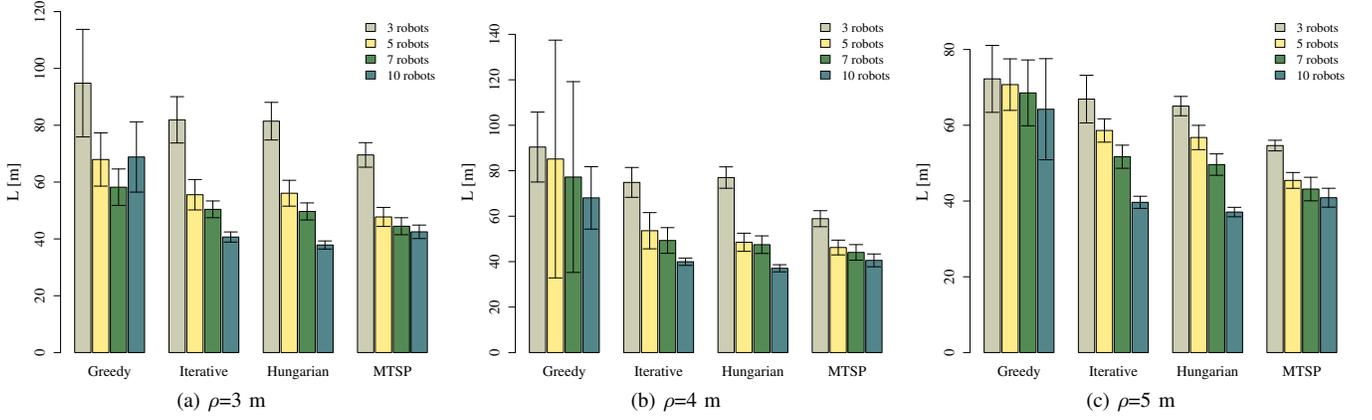


Fig. 2. Scaling of the exploration strategy with the increasing number of robots and for laser range ρ .

depends on the initial conditions, i.e., a small perturbation leads to significantly different performance. This is not the case of the other methods, and especially for the MTSP, which provide more stable solutions as s_L is low.

TABLE II
COMPARISON OF THE MRE STRATEGIES

ρ	m	Iterative	Hungarian	MTSP
		vs Greedy	vs Iterative	vs Hungarian
3.0	3	+	=	+
3.0	5	+	=	+
3.0	7	+	=	+
3.0	10	+	+	-
4.0	3	+	=	+
4.0	5	+	=	=
4.0	7	+	=	+
4.0	10	+	+	-
5.0	3	+	=	+
5.0	5	+	=	+
5.0	7	+	=	+
5.0	10	+	+	-

1) *Influence of the planning period:* The influence of s_{max} to the performance of the exploration is depicted in Fig. 3. The results have been obtained for $m=7$ and $\rho=3$ m, and for each value of s_{max} all problem variants have been considered as well as 20 trials for each problem variant and the stochastic strategy. The total number of the performed trials in this evaluation is 14 280. The results indicate that a smaller value of s_{max} generally provides better results, but for all tested values of s_{max} the proposed MTSP method provides superior results. Regarding the required computational time the simple greedy or iterative strategies are computationally less intensive, but due to a longer exploration time, all the methods are competitive in the total required computational time.

B. Results using real navigational framework

Performance of the tested exploration strategies have also been evaluated using Player/Stage framework, in which additional components of the navigational architecture play role. The robot configuration and the environment is same as in

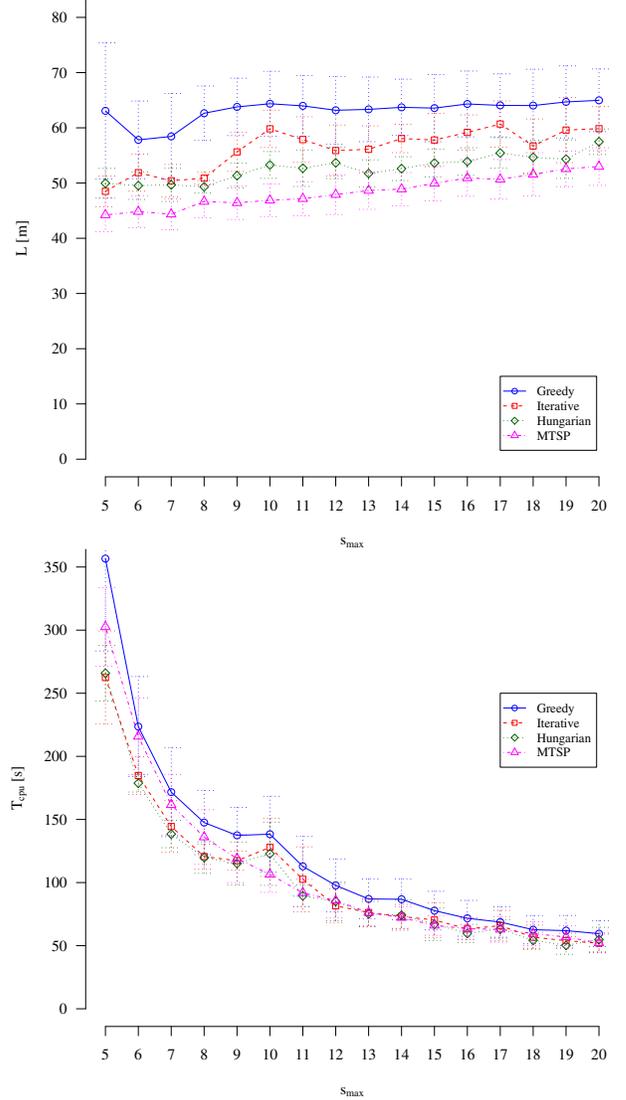


Fig. 3. Influence of the planning period s_{max} to the exploration performance for $m=7$ and $\rho=3$ m; *upper*: average values of the maximal travelled distance; *bottom*: average values of the required computational times using C++ and a workstation with 3.2 GHz CPU running FreeBSD 9.

the previous tests. The main difference is that a robot is controlled using the SND driver [19] for the robot motion and obstacle avoidance.

TABLE III

PERFORMANCE OF THE STRATEGIES USING PLAYER/STAGE AND $\rho=3$ m

m	Greedy		Iterative		Hungarian		MTSP	
	L	s_L	L	s_L	L	s_L	L	s_L
3	73.8	9.8	66.5	4.6	65.2	4.8	56.0	7.1
5	55.5	9.3	47.0	8.2	49.2	9.0	49.2	7.0

In this test, the replanning frequency has not been restricted, i.e., it is limited only by the hardware used. Therefore, the presented results provide estimation of the real benefits of more computational demanding methods over simple and faster methods. The results are shown in Table III.

C. Discussion

The presented results indicate that the proposed MTSP method provides more efficient tasks allocation than the former approaches. Hence, the results support the idea to consider a longer planning horizon rather than just an immediate goal. However, for ten robots the benefit of the method is not evident from the presented results. It is probably due to a relatively small environment and the used clustering initialized by the robots' position, which can lead to few dominant clusters and a greedy assignment.

Performance of the Hungarian and Iterative strategies is very similar, therefore, the main advantage of the Iterative strategy is its simpler implementation. Besides, the Iterative strategy can also be easily deployed in a distributed environment, which is not the case of the Hungarian algorithm. On the other hand, computational requirements of the more sophisticated Hungarian and MTSP approaches are competitive to the simple greedy algorithm; thus, they should be preferred in the applicable scenarios.

During the experimental evaluation, we have noticed, the navigational framework, in particular the local planner, affects the performance of the exploration. This is mostly visible in a situation where a robot is approaching a narrow passage (e.g., doors), where its velocity is slow. Differences in the robot average velocities affect the total required time. Therefore, the expected distance cost to reach the goal is only approximation, which in consequence means that the exploration strategy does not provide the expected benefit.

VII. CONCLUSION

In this paper, we further developed our previous work on exploration strategy using the TSP distance cost to the multi-robot exploration. The proposed strategy is compared with three standard approaches and the results show the proposed novel multi-robot exploration provides better results while its total computational requirements are competitive. Although only relatively small number of robots has been considered, the results indicated that for a higher number of robots the Iterative and Hungarian algorithms provides similar results to the proposed MTSP based strategy.

Regarding the found insights, the real performance of exploration is not affected only by the used strategy, but also by the low-level motion control. Therefore, we are aiming to consider more realistic estimation of the travelling cost towards the goal in the assignment problem. Besides, we

also intend to evaluate different exploration strategies using real robots to verify the results presented in this paper.

ACKNOWLEDGMENTS

This work has been supported by the Technology Agency of the Czech Republic under Project No. TE01020197 and by the Ministry of Education of the Czech Republic under Project No. LH11053.

The access to computing and storage facilities provided under the National Grid Infrastructure MetaCentrum, provided under project No. LM2010005 funded by Ministry of Education of Czech Republic is highly appreciated.

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